Дис 6.12.12

Намерете производната

$$f(x) = \frac{\left(\operatorname{arctg}\frac{1}{x}\right)^2}{\operatorname{tg}\frac{1}{x}}$$

Св-ва на производната

$$(f+g)' = f' + g' \quad ; \quad (f \circ g)' = f' \circ g \cdot g'$$

$$(fg)' = f'g + g'f \quad ; \quad (f \circ g)(x) \overset{\text{def}}{=} f(g(x))$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

$$\left(f^{-1}(f(x))\right)' = \frac{1}{f'(x)} \qquad f'(x) \neq 0$$

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2} \quad , \quad (\operatorname{tg} x)' = \frac{1}{\cos^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$g(x) = \operatorname{tg} x = y \qquad x = g^{-1}y = \operatorname{arctg} y \qquad (g^{-1}(y))' = \frac{1}{g'(x)} = \frac{1}{\frac{1}{\cos^2 x}} = \cos^2 x$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} \quad , \quad \cos x = \frac{\sin x}{\operatorname{tg} x} \quad , \quad \cos^2 = \frac{1}{1+\operatorname{tg}^2 x} = \frac{1}{1+y^2}$$

$$\left(\operatorname{arctg} \tfrac{1}{x}\right)' = \frac{1}{1+\left(\frac{1}{x}\right)^2}\left(\frac{1}{x}\right)' = \frac{x^2}{x^2+1}\left(\frac{-1}{x^2}\right) = -\frac{1}{x^2+1}$$

Разглеждаме $\dfrac{\left(\operatorname{arctg}\frac{1}{x}\right)^2}{\operatorname{tg}\frac{1}{x}}$ и търсим производна

$$f'(x) = \frac{2\operatorname{arctg}\frac{1}{x}\cdot\left(\operatorname{arctg}\frac{1}{x}\right)'\cdot \operatorname{tg}\frac{1}{x} - \frac{1}{\cos^2\frac{1}{x}}\left(\frac{1}{x}\right)'\left(\operatorname{arctg}\frac{1}{x}\right)^2}{\operatorname{tg}^2\frac{1}{x}}$$

$$\frac{2\operatorname{arctg}\frac{1}{x}\,\frac{(-1)}{1+x^2}\operatorname{tg}\frac{1}{x} + \frac{1}{x-2x^2\cos^2}\operatorname{arctg}^2\frac{1}{x}}{\operatorname{tg}^2\frac{1}{x}}$$

$$f(x) = \begin{cases} \sin\frac{1}{x} & \text{при } x \neq 0 \\ 0 & \text{при } x = 0 \end{cases} \qquad x \in \mathbb{R}$$

$?\ f'(x)$

$$f'(x) = \left(\sin\frac{1}{x}\right)' = \cos\frac{1}{x}\left(-\frac{1}{x^2}\right) \text{ за } x \neq 0$$

$$f'(0) \overset{\text{def}}{=} \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\sin\frac{1}{h}}{h}$$

$$\lim_{h\to 0} \frac{1}{h}\sin\frac{1}{h} \Rightarrow f'(0) \nexists$$

$\frac{1}{h} \to \infty$

разглеждане $\sin^2 x \Rightarrow \lim_{h\to 0} \frac{h^2 \sin\frac{1}{h}}{h} =$

$$\lim_{h\to 0} h\sin\frac{1}{h} \Rightarrow f'(x) \to 0$$

когато $h \to 0$ т. е. имаме

$$0 \cdot \sin\frac{1}{h} = 0$$

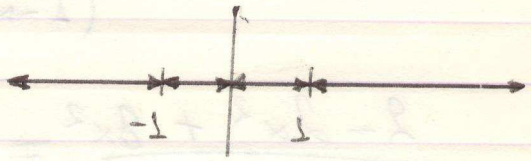
$f(x) = \frac{1}{1-x^2}$ първо определяме ДМ

$ДМ = \{x \in R \mid x \neq \pm 1\}$

Разглеждаме за $x > 0$

$f(-x) = f(x)$

$\lim\limits_{x \to +\infty} \frac{1}{1-x^2} \to 0$

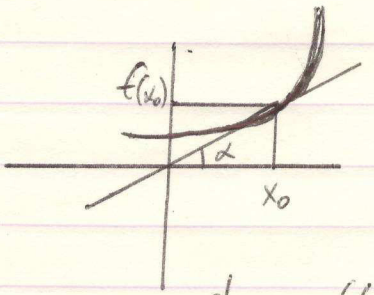


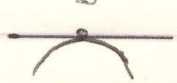
$\lim\limits_{x \to 1^+} \frac{1}{1-x^2} \longrightarrow -\infty$

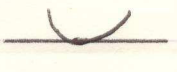
$\lim\limits_{x \to 1^-} \frac{1}{1-x^2} \longrightarrow +\infty$

Деф

Графика на $f: X \to Y$ се нарича множеството $\Gamma_f = \{X f(x)\} \in (X+Y) \mid x \in X\}$




изпъкнал


вдлъбнат

$f'(x) = \left((1-x^2)^{-1}\right)' = -1(1-x^2)^{-2}(-2x) =$

$\frac{2x}{(1-x^2)^2}$

f расте ако $f' > 0$ строго

f намалява ако $f' < 0$ строго

Разглеждаме

$f(x) \Leftrightarrow \frac{2x}{(1-x^2)^2} > 0 \Rightarrow x > 0$

и $\frac{2x}{(1-x^2)^2} < 0 \Rightarrow x < 0 \quad x \in DM$

защото x^2 не си мени знака
но остава знака на $2x$

$$f''(x) = \frac{2(1-x^2)^2 - 2(1-x^2)(-2x)(2x)}{(1-x^2)^{4\,3}} \Rightarrow$$

$$\frac{2-2x^2+8x^2}{(1-x^2)^3} = \frac{6x^2+2}{(1-x^2)^3}$$

f е изпъкнала ако $f'' > 0$ и вдлъбната ако $f'' < 0$

$f'' > 0 \Leftrightarrow x < 1$

$f'' < 0 \Leftrightarrow x > 1$